

Gravitational Frames and Scalar Field Dynamics

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Abstract

Scalar fields describe interesting phenomena such as Higgs bosons, dark matter and dark energy, and are found to be quite common in physical theories. These fields are susceptible to gravitational forces so that being massless is not enough to remain conformal invariant. They should also be connected directly to the scalar curvature. Because of this characteristics, we investigated the structure and interactions of scalar fields under the conformal transformations. We show how to reduce the quadratic quantum contributions in the single scalar field theory. In the multi-scalar field theories, we analyzed interactions in certain limits. We suggest a new method for stabilizing Higgs bosons.

1 Introduction

After the construction of general theory of relativity on Riemann geometry by Einstein [1], the metric tensor which is the same in everywhere of manifold became a basic geometric entitle. Soon, with the analogy of gauge transformation of electromagnetism, the idea of gauge transformation of geometry proposed by Weyl [2, 3]. This idea was progressed by Cartan in different studies [4, 5, 6, 7]. Thus, the idea of two metric tensor of different manifold connected via a conformal function was introduced by Weyl [3]. Then, the conformal transformation accumulated in different studies in different context [8, 9, 10, 11, 12, 13]. For more information one can see the [14], and references there in. Recently, Demir [15] showed that the Higgs field could be transferred from weak field to the gravitational field via a conformal way. In general, scalar fields can be transferred in metric sector by conformal way.

Experiments are still showing that the most working model for nature is the Standard Model (SM) [16]. After the discovery of Higgs boson, one missing part of the SM (electromagnetic, weak and strong interactions) has been completed. Since the Higgs field is scalar, naturally the quantum corrections of scalar fields are unstable [17]. On the other hand, the experimental evidences indicate the SM as more representing candidate of nature. This indication leads us to the elimination instability of scalar fields, which can be done using a conformal transformation. Nobili [18] show that the dilaton fields, which is geometry based and vacuum expectation value of them are not zero, can transfer energy

from geometry to the matter sector by interacting with the massless ordinary scalar fields, which has positive energy. This interaction produces Higgs field. Thus, one important candidate to combine the SM and general relativity (GR) is the conformal general relativity (CGR).

In our study we show that unstable scalar field is to be stabilized by conformal way. This procedure is a new method for stabilizing Higgs fields. In addition we analyzed two-scalar field in certain limits using same method.

This paper is organized as follows. First section contains the motivation and notation of the work. Second section contains the Lagrangian with a scalar field and its conformal transformation. Third section contains Lagrangian with two-scalar fields with certain limits used. The conclusion is given in the last section.

2 Notation and motivation

After the idea of fields can attain mass via spontaneous symmetry breaking mechanism, the CGR attracts a new interest. Two metric fields, $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$, which belong to different manifolds can be related with smooth, strictly positive function, *i.e.* conformal function $\Omega(x)$ as follows:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}. \quad (1)$$

The inverse is

$$\tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}, \quad (2)$$

and its determinant is

$$\tilde{g} = \Omega^8 g. \quad (3)$$

The metric signature is $\text{diag}(-, +, +, +)$. In this regard, the conformal transformation of Christoffel connection, Riemann tensor, Ricci tensor, and Ricci scalar are respectively as follows [19].

$$\tilde{\Gamma}_{\mu\nu}^{\alpha} = \Gamma_{\mu\nu}^{\alpha} + 2\delta_{(\mu}^{\alpha} \nabla_{\nu)} \ln \Omega - g_{\mu\nu} g^{\alpha\beta} \nabla_{\beta} \ln \Omega, \quad (4)$$

$$\begin{aligned} \tilde{R}_{\mu\nu\alpha}^{\lambda} = & R_{\mu\nu\alpha}^{\lambda} + 2\delta_{[\mu}^{\lambda} \nabla_{\nu]} \nabla_{\alpha} \ln \Omega - 2g^{\lambda\beta} g_{\alpha[\mu} \nabla_{\nu]} \nabla_{\beta} \ln \Omega + \\ & 2(\nabla_{[\mu} \ln \Omega) \delta_{\nu]}^{\lambda} \nabla_{\alpha} \ln \Omega - 2(\nabla_{[\mu} \ln \Omega) g_{\nu]\alpha} g^{\lambda\rho} \nabla_{\rho} \ln \Omega - \\ & 2g_{\alpha[\mu} \delta_{\nu]}^{\lambda} g_{\beta\rho} \nabla_{\rho} \ln \Omega, \end{aligned} \quad (5)$$

$$\begin{aligned} \tilde{R}_{\mu\alpha} = & R_{\mu\alpha} - (n-2) \nabla_{\mu} \nabla_{\alpha} \ln \Omega - g_{\mu\alpha} g^{\lambda\rho} \nabla_{\lambda} \nabla_{\rho} \ln \Omega + \\ & (n-2)(\nabla_{\mu} \ln \Omega) \nabla_{\alpha} \ln \Omega - (n-2) g_{\mu\alpha} g^{\lambda\rho} \nabla_{\lambda} \ln \Omega \nabla_{\rho} \ln \Omega, \end{aligned} \quad (6)$$

$$\tilde{R} = \Omega^{-2} \{ R - 2(n-1)g^{\mu\alpha}\nabla_\mu\nabla_\alpha \ln\Omega - (n-2)(n-1)g^{\mu\alpha}\nabla_\mu \ln\Omega \nabla_\alpha \ln\Omega \}. \quad (7)$$

Where $\tilde{R} = \tilde{g}^{\mu\nu} \tilde{R}_{\mu\nu}$.

3 Conformal transformation in tensor-scalar fields

An action of general theory with a scalar field can be written as follows

$$S(\tilde{g}, \tilde{\phi}) = \int d^4x \sqrt{-\tilde{g}} \{ \tilde{C}(\tilde{\phi}) R(\tilde{g}) + \tilde{K}(\tilde{\phi}) \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \tilde{V}(\tilde{\phi}) \}. \quad (8)$$

Where \tilde{V} is potential and given by

$$\tilde{V}(\tilde{\phi}) = \tilde{V}_0 + \frac{1}{2} \tilde{m}^2 \tilde{\phi}^2 + \frac{1}{4} \tilde{\lambda} \tilde{\phi}^4, \quad (9)$$

\tilde{C} is conformal coupling given by

$$\tilde{C}(\tilde{\phi}) = \frac{1}{2} (\tilde{c} \tilde{\phi}^2 + \tilde{M}_g^2), \quad (10)$$

and \tilde{K} are functional coefficients,

$$\tilde{K}(\tilde{\phi}) = \tilde{k} \quad (11)$$

\tilde{M}_g in equation (10) is the gravitational mass. Under the

$$\Omega(x) = \frac{M_c}{\phi}, \quad (12)$$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}, \quad (13)$$

transformation, the action (8) becomes

$$\begin{aligned} S(g, \phi) = \int d^4x \sqrt{-g} \{ & \frac{1}{2} (\tilde{c} M_c^2 + (\frac{\tilde{M}_g}{M_c})^2 \phi^2) R + [3(\frac{\tilde{M}_g}{M_c})^2 + \\ & (\tilde{k} - 3\tilde{c})(\frac{M_c}{\phi})^2] g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{4} \tilde{\lambda} M_c^4 - \frac{1}{2} \tilde{m}^2 \phi^2 - \frac{\tilde{V}_0}{M_c^4} \phi^4 \}. \end{aligned} \quad (14)$$

Where M_c is the dimensional mass parameter. Setting $\tilde{c} M_c^2 \equiv M_{pl}^2$, $(\frac{\tilde{M}_g}{M_c})^2 \equiv -\xi_c$, and $3\tilde{c} = \tilde{k}$, equation (14) becomes

$$S(g, \phi) = \int d^4x \sqrt{-g} \{ \frac{1}{2} [(M_{pl}^2 - \xi_c \phi^2) R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi] - V(\phi) \}. \quad (15)$$

Where

$$V(\phi) = \frac{1}{4}\tilde{\lambda}M_c^4 + \frac{1}{2}\tilde{m}^2\phi^2 + \frac{\tilde{V}_0}{M_c^4}\phi^4, \quad (16)$$

$\xi_c = \frac{1}{6} = \tilde{c} = \xi_c$, $\tilde{M}_g^2 = -M_{pl}^2$, and $\phi = M_c^2/\tilde{\phi}$. This conformal transformation has two properties; first, transformation function Ω is related to the ϕ . Second, scalar field ϕ transforms as $\phi = M_c^2/\tilde{\phi}$.

These two properties are emerging very important results:

1. From equation (9), the vacuum term \tilde{V}_0 , transforms to the four-coupling parameter, $\tilde{\lambda}$ in equation (16).
2. Four-coupling parameter $\tilde{\lambda}$ in equation (9), gives the vacuum energy \tilde{V}_0 in equation (16).
3. This conformal transformation does not change the scalar field mass, $\tilde{m}^2 = m^2$.
4. ϕ stays as a real scalar field, while $\tilde{\phi}$ is the ghost field.
5. From the transformation properties; even if equation (8) has no scalar field, Ω treats as a new scalar field and transforms the gravitational theory to a scalar tensor theory.

These properties give a new way to overcome the problems of stabilizing scalar field theories. The first one is the huge quantum corrections to the scalar fields masses. In literature, *e.g.* in [20], quantum correction of a scalar field mass is given as

$$\delta\tilde{m}^2 \propto \frac{\tilde{\lambda}}{(4\pi)^2}\Lambda^2 \quad (17)$$

Λ is the most reachable energy-momentum scale. In general it is proportional to the M_{pl} . Quantum correction of mass is very huge, and thus the theory become trivial at the quantum level. This phenomena works also for Higgs boson. But, under the conformal transformation the quantum correction of (17) can be written as follows

$$\delta m^2 \propto \frac{\lambda}{(4\pi)^2}\Lambda^2 \quad (18)$$

via a simple calculation. In this equation the λ is equal to the $\tilde{V}_0/9M_{pl}^4$. This value is restricted with the vacuum energy of equation (9), \tilde{V}_0 . Thus, the value of δm^2 can be reduced accordingly. In general, ϕ could be steady by setting $\tilde{V}_0 \ll M_{pl}^4$. For example if we set $\tilde{V}_0 \cong m^2 M_{pl}^4/\Lambda^2$, the mass of ϕ will be stabilized. With this mechanism (conformal transformation) the scalar field mass is being stabilized by gravitational force. Because of Higgs field is scalar field, this transformation also works for stabilizing Higgs boson mass.

4 Multi-scalar fields

The procedure which is performed in the last section, can be applied to the multi scalar fields theories. For instance, a general theory with two scalar fields

can be written as follows.

$$S(\tilde{g}, \tilde{\phi}) = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{1}{2} [(\tilde{M}_g^2 + \tilde{c}_1 \tilde{\phi}_1^2 + \tilde{c}_2 \tilde{\phi}_2^2) \tilde{R} - \tilde{k}_1 \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi}_1 \partial_\nu \tilde{\phi}_1 - \tilde{k}_2 \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi}_2 \partial_\nu \tilde{\phi}_2 - \tilde{k}_{12} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi}_1 \partial_\nu \tilde{\phi}_2] - \tilde{V}(\tilde{\phi}_1, \tilde{\phi}_2) \right\}. \quad (19)$$

Where $\tilde{k}_1, \tilde{k}_2, \tilde{k}_{12}, \tilde{c}_1$, and \tilde{c}_2 are coupling constants, and \tilde{V} is the potential determined as

$$\tilde{V}(\tilde{\phi}_1, \tilde{\phi}_2) = \tilde{V}_0 + \frac{1}{2} \tilde{m}_1^2 \tilde{\phi}_1^2 + \frac{1}{2} \tilde{m}_2^2 \tilde{\phi}_2^2 + \frac{1}{4} \tilde{\lambda}_1 \tilde{\phi}_1^4 + \frac{1}{4} \tilde{\lambda}_2 \tilde{\phi}_2^4 + \frac{1}{4} \tilde{\lambda}_{12} \tilde{\phi}_1^2 \tilde{\phi}_2^2. \quad (20)$$

Under the

$$\tilde{g}^{\mu\nu} = \left(\frac{\tilde{\phi}_1}{M_{c_1}} \right)^{\alpha_1} \left(\frac{\tilde{\phi}_2}{M_{c_2}} \right)^{\alpha_2} g^{\mu\nu}, \quad (21)$$

transformation and setting

$$\tilde{\phi}_1 = M_{c_1}^2 / \phi_1, \quad (22)$$

$$\tilde{\phi}_2 = M_{c_2}^2 / \phi_2, \quad (23)$$

the equation (19) becomes

$$S(g, \phi) = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} [C(\phi_1, \phi_2) R + K_1(\phi_1, \phi_2) g^{\mu\nu} \partial_\mu \phi_1 \partial_\nu \phi_1 + K_2(\phi_1, \phi_2) g^{\mu\nu} \partial_\mu \phi_2 \partial_\nu \phi_2 + K_{12}(\phi_1, \phi_2) g^{\mu\nu} \partial_\mu \phi_1 \partial_\nu \phi_2] - V(\phi_1, \phi_2) \right\}. \quad (24)$$

Where α_1 , and α_2 are arbitrary constants. Functional coefficients C , K_1 , K_2 and K_{12} , and potential $V(\phi_1, \phi_2)$ in equation (24) are defined as follows:

$$C(\phi_1, \phi_2) = \tilde{M}_g^2 \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2} + \tilde{c}_1 M_{c_1}^2 \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1-2} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2} + \tilde{c}_2 M_{c_2}^2 \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2-2} \quad (25)$$

$$K_1(\phi_1, \phi_2) = \frac{3}{2} [\alpha_1^2 \left(\frac{\tilde{M}_g}{M_{c_1}} \right)^2 \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1-2} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2} + \tilde{c}_1 \alpha_1 (\alpha_1 + 4) \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2} + \tilde{c}_2 \alpha_1^2 \left(\frac{M_{c_2}}{M_{c_1}} \right)^2 \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1-2} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2+2}] - \tilde{k}_1 \left(\frac{\phi_1}{M_{c_1}} \right)^{\alpha_1-4} \left(\frac{\phi_2}{M_{c_2}} \right)^{\alpha_2}, \quad (26)$$

$$\begin{aligned}
K_2(\phi_1, \phi_2) = & \frac{3}{2}[\alpha_2^2(\frac{\widetilde{M}_g}{M_{c_2}})^2(\frac{\phi_1}{M_{c_1}})^{\alpha_1}(\frac{\phi_2}{M_{c_2}})^{\alpha_2-2} + \\
& \widetilde{c}_2\alpha_2(\alpha_2+4)(\frac{\phi_1}{M_{c_1}})^{\alpha_1}(\frac{\phi_2}{M_{c_2}})^{\alpha_2} + \\
& \widetilde{c}_1\alpha_2^2(\frac{M_{c_1}}{M_{c_2}})^2(\frac{\phi_1}{M_{c_1}})^{\alpha_1+2}(\frac{\phi_2}{M_{c_2}})^{\alpha_2-2}] - \widetilde{k}_2(\frac{\phi_1}{M_{c_1}})^{\alpha_1}(\frac{\phi_2}{M_{c_2}})^{\alpha_2-4}, \quad (27)
\end{aligned}$$

$$\begin{aligned}
K_{12}(\phi_1, \phi_2) = & 3[\frac{\alpha_1\alpha_2\widetilde{M}_g^2}{M_{c_1}M_{c_2}}(\frac{\phi_1}{M_{c_1}})^{\alpha_1-1}(\frac{\phi_2}{M_{c_2}})^{\alpha_2-1} + \\
& \widetilde{c}_2\alpha_1(\alpha_2+2)(\frac{M_{c_2}}{M_{c_1}})(\frac{\phi_1}{M_{c_1}})^{\alpha_1-1}(\frac{\phi_2}{M_{c_2}})^{\alpha_2+1} + \\
& \widetilde{c}_1\alpha_2(\alpha_1+2)(\frac{M_{c_1}}{M_{c_2}})(\frac{\phi_1}{M_{c_1}})^{\alpha_1+1}(\frac{\phi_2}{M_{c_2}})^{\alpha_2-1}] - \widetilde{k}_{12}(\frac{\phi_1}{M_{c_1}})^{\alpha_1-2}(\frac{\phi_2}{M_{c_2}})^{\alpha_2-2}, \quad (28)
\end{aligned}$$

and

$$\begin{aligned}
V(\phi_1, \phi_2) = & \widetilde{V}_0(\frac{\phi_1}{M_{c_1}})^{2\alpha_1}(\frac{\phi_2}{M_{c_2}})^{2\alpha_2} + \frac{1}{2}\widetilde{m}_1^2M_{c_1}^2(\frac{\phi_1}{M_{c_1}})^{2\alpha_1-2}(\frac{\phi_2}{M_{c_2}})^{2\alpha_2} + \\
& \frac{1}{2}\widetilde{m}_2^2M_{c_2}^2(\frac{\phi_1}{M_{c_1}})^{2\alpha_1}(\frac{\phi_2}{M_{c_2}})^{2\alpha_2-2} + \frac{1}{4}\widetilde{\lambda}_1M_{c_1}^4(\frac{\phi_1}{M_{c_1}})^{2\alpha_1-4}(\frac{\phi_2}{M_{c_2}})^{2\alpha_2} + \\
& \frac{1}{4}\widetilde{\lambda}_2M_{c_2}^4(\frac{\phi_1}{M_{c_1}})^{2\alpha_1}(\frac{\phi_2}{M_{c_2}})^{2\alpha_2-4} + \frac{1}{4}\widetilde{\lambda}_{12}M_{c_1}^2M_{c_2}^2(\frac{\phi_1}{M_{c_1}})^{2\alpha_1-2}(\frac{\phi_2}{M_{c_2}})^{2\alpha_2-2}. \quad (29)
\end{aligned}$$

The values of these functional coefficients and the potential are very different for different values of α . Some cases are calculated as follows:

Case 1 $\alpha_1 = 1$, and $\alpha_2 = 0$:

$$C(\phi_1, \phi_2) = \widetilde{M}_g^2(\frac{\phi_1}{M_{c_1}}) + \widetilde{c}_1M_{c_1}^2(\frac{M_{c_1}}{\phi_1})(\frac{\phi_2}{M_{c_2}}) + \widetilde{c}_2M_{c_2}^2(\frac{\phi_1}{M_{c_1}})(\frac{M_{c_2}}{\phi_2})^2 \quad (30)$$

$$K_1(\phi_1, \phi_2) = \frac{3}{2M_{c_1}}[5\widetilde{c}_1\phi_1 + \widetilde{c}_2\frac{\phi_2^2}{\phi_1} + \frac{\widetilde{M}_g^2}{\phi_1}] - \widetilde{k}_1(\frac{M_{c_1}}{\phi_1})^3 \quad (31)$$

$$K_2(\phi_1, \phi_2) = -\widetilde{k}_2\frac{\phi_1}{M_{c_1}}(\frac{M_{c_2}}{\phi_2})^4 \quad (32)$$

$$K_{12}(\phi_1, \phi_2) = 6\widetilde{c}_2\frac{\phi_2}{M_{c_1}} - \widetilde{k}_{12}(\frac{M_{c_1}}{\phi_1})(\frac{M_{c_2}}{\phi_2})^2 \quad (33)$$

$$V(\phi_1, \phi_2) = V_0 + \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}\tilde{m}_2^2M_{c_2}^2\left(\frac{\phi_1}{M_{c_1}}\right)^2\left(\frac{M_{c_2}}{\phi_2}\right)^2 + \frac{1}{4}\tilde{\lambda}_1M_{c_1}^4\left(\frac{M_{c_1}}{\phi_1}\right)^2 + \frac{1}{4}\tilde{\lambda}_2M_{c_2}^4\left(\frac{\phi_1}{M_{c_1}}\right)^2\left(\frac{M_{c_2}}{\phi_2}\right)^4 + \frac{1}{4}\tilde{\lambda}_{12}M_{c_1}^2M_{c_2}^2\left(\frac{M_{c_2}}{\phi_2}\right)^2. \quad (34)$$

Where $V_0 = \frac{1}{2}\tilde{m}_1^2M_{c_1}^2$, the transformed vacuum energy, and $m_1^2 = \frac{2\tilde{V}_0}{M_{c_1}^2}$. Untransformed vacuum energy \tilde{V}_0 determines the transformed scalar field ϕ_1 mass, m_1 .

Case 2 $\alpha_1 = 1$, and $\alpha_2 = 1$:

$$C(\phi_1, \phi_2) = \tilde{M}_g^2\left(\frac{\phi_1}{M_{c_1}}\right)\left(\frac{\phi_2}{M_{c_2}}\right) + \tilde{c}_1M_{c_1}^2\left(\frac{M_{c_1}}{\phi_1}\right)\left(\frac{\phi_2}{M_{c_2}}\right) + \tilde{c}_2M_{c_2}^2\left(\frac{\phi_1}{M_{c_1}}\right)\left(\frac{M_{c_2}}{\phi_2}\right) \quad (35)$$

$$K_1(\phi_1, \phi_2) = \frac{3}{2M_{c_1}M_{c_2}}[5\tilde{c}_1\phi_1\phi_2 + \tilde{c}_2\frac{\phi_2^3}{\phi_1} + \tilde{M}_g^2\frac{\phi_2}{\phi_1}] - \tilde{k}_1\frac{\phi_2}{M_{c_2}}\left(\frac{M_{c_1}}{\phi_1}\right)^3 \quad (36)$$

$$K_2(\phi_1, \phi_2) = \frac{3}{2M_{c_1}M_{c_2}}[5\tilde{c}_2\phi_1\phi_2 + \tilde{c}_1\frac{\phi_1^3}{\phi_2} + \tilde{M}_g^2\frac{\phi_1}{\phi_2}] - \tilde{k}_2\frac{\phi_1}{M_{c_1}}\left(\frac{M_{c_2}}{\phi_2}\right)^3 \quad (37)$$

$$K_{12}(\phi_1, \phi_2) = \frac{3}{M_{c_1}M_{c_2}}[\tilde{M}_g^2 + 3\tilde{c}_2\phi_2^2 + 3\tilde{c}_1\phi_1^2] - \tilde{k}_{12}\frac{M_{c_1}}{\phi_1}\frac{M_{c_2}}{\phi_2} \quad (38)$$

$$V(\phi_1, \phi_2) = V_0 + \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}m_2^2\phi_2^2 + \frac{1}{4}\tilde{\lambda}_1M_{c_1}^4\left(\frac{M_{c_1}}{\phi_1}\right)^2\left(\frac{\phi_2}{M_{c_2}}\right)^2 + \frac{1}{4}\tilde{\lambda}_2M_{c_2}^4\left(\frac{\phi_1}{M_{c_1}}\right)^2\left(\frac{M_{c_2}}{\phi_2}\right)^2 + \frac{1}{4}\lambda_{12}\phi_1^2\phi_2^2. \quad (39)$$

Where $\lambda_{12} = \frac{4\tilde{V}_0}{(M_{c_1}M_{c_2})^2}$, and $V_0 = \frac{1}{4}\tilde{\lambda}_{12}(M_{c_1}M_{c_2})^2$. The coupling constant, $\tilde{\lambda}_{12}$, is transformed to the vacuum energy, V_0 , and the vacuum energy, \tilde{V}_0 , is transformed to the coupling constant λ_{12} . The scalar field masses are transformed as follows

$$m_1^2 = \tilde{m}_2^2\left(\frac{M_{c_2}}{M_{c_1}}\right)^2, \quad (40)$$

$$m_2^2 = \tilde{m}_1^2\left(\frac{M_{c_1}}{M_{c_2}}\right)^2. \quad (41)$$

The scalar field masses are related to the $\tilde{m}_1, \tilde{m}_2, M_{c_1}$, and M_{c_2} .

Case 3 $\alpha_1 = 2$, and $\alpha_2 = 0$:

$$C(\phi_1, \phi_2) = \widetilde{M}_g^2 \left(\frac{\phi_1}{M_{c_1}} \right)^2 + \widetilde{c}_1 M_{c_1}^2 + \widetilde{c}_2 M_{c_2}^2 \left(\frac{\phi_1}{M_{c_1}} \right)^2 \left(\frac{M_{c_2}}{\phi_2} \right)^2 \quad (42)$$

$$K_1(\phi_1, \phi_2) = \frac{6}{M_{c_1}^2} [3\widetilde{c}_1 \phi_1^2 + \widetilde{M}_g^2 + \widetilde{c}_2 \phi_2^2] - \widetilde{k}_1 \left(\frac{M_{c_1}}{\phi_1} \right)^2 \quad (43)$$

$$K_2(\phi_1, \phi_2) = -\widetilde{k}_2 \left(\frac{\phi_1}{M_{c_1}} \right)^2 \left(\frac{M_{c_2}}{\phi_2} \right)^4 \quad (44)$$

$$K_{12}(\phi_1, \phi_2) = \frac{12}{M_{c_1}^2} \phi_1 \phi_2 - \widetilde{k}_{12} \left(\frac{M_{c_2}}{\phi_2} \right)^2 \quad (45)$$

$$V(\phi_1, \phi_2) = V_0 + \frac{1}{2} m_1^2 \phi_1^2 + \frac{1}{2} \widetilde{m}_2^2 M_{c_2}^2 \left(\frac{\phi_1}{M_{c_1}} \right)^4 \left(\frac{M_{c_2}}{\phi_2} \right)^2 + \frac{1}{4} \lambda_1 \phi_1^4 + \frac{1}{4} \widetilde{\lambda}_2 M_{c_2}^4 \left(\frac{\phi_1}{M_{c_1}} \right)^4 \left(\frac{M_{c_2}}{\phi_2} \right)^4 + \frac{1}{4} \widetilde{\lambda}_{12} M_{c_2}^4 \left(\frac{\phi_1}{\phi_2} \right)^2. \quad (46)$$

Where $\lambda_1 = \frac{4\widetilde{V}_0}{M_{c_1}^4}$, $V_0 = \frac{1}{4} \widetilde{\lambda}_1 M_{c_1}^4$, and $\widetilde{m}_1 = m_1$.

Case 4 $\alpha_1 = 2$, and $\alpha_2 = 2$:

$$C(\phi_1, \phi_2) = \widetilde{M}_g^2 \left(\frac{\phi_1}{M_{c_1}} \right)^2 \left(\frac{\phi_2}{M_{c_2}} \right)^2 + c_1 \phi_1^2 + c_2 \phi_2^2 \quad (47)$$

$$K_1(\phi_1, \phi_2) = \frac{6}{M_{c_1}^2 M_{c_2}^2} [\widetilde{M}_g^2 \phi_2^2 + 3\widetilde{c}_1 \phi_1^2 \phi_2^2 + \widetilde{c}_2 \phi_2^4] - \widetilde{k}_1 \left(\frac{M_{c_1} \phi_2}{M_{c_2} \phi_1} \right)^2 \quad (48)$$

$$K_2(\phi_1, \phi_2) = \frac{6}{M_{c_1}^2 M_{c_2}^2} [\widetilde{M}_g^2 \phi_1^2 + 3\widetilde{c}_2 \phi_1^2 \phi_2^2 + \widetilde{c}_1 \phi_1^4] - \widetilde{k}_2 \left(\frac{M_{c_2} \phi_1}{M_{c_1} \phi_2} \right)^2 \quad (49)$$

$$K_{12}(\phi_1, \phi_2) = \frac{12}{M_{c_1}^2 M_{c_2}^2} [\widetilde{M}_g^2 \phi_1 \phi_2 + 2\widetilde{c}_2 \phi_1 \phi_2^3 + 2\widetilde{c}_1 \phi_2 \phi_1^3] - \widetilde{k}_{12} \quad (50)$$

$$V(\phi_1, \phi_2) = V_0 + \frac{1}{2} m_1^2 \phi_1^2 + \frac{1}{2} m_2^2 \phi_2^2 + \frac{1}{4} \lambda_1 \phi_1^4 + \frac{1}{4} \lambda_2 \phi_2^4 + \frac{1}{4} \lambda_{12} \phi_1^2 \phi_2^2. \quad (51)$$

Where $V_0 = \widetilde{V}_0 \left(\frac{\phi_1}{M_{c_1}} \right)^4 \left(\frac{\phi_2}{M_{c_2}} \right)^4$, $c_2 = \widetilde{c}_1 \left(\frac{M_{c_1}}{M_{c_2}} \right)^2$, $c_1 = \widetilde{c}_2 \left(\frac{M_{c_2}}{M_{c_1}} \right)^2$, $\lambda_2 = \widetilde{\lambda}_1 \left(\frac{M_{c_1}}{M_{c_2}} \right)^4$, $\lambda_1 = \widetilde{\lambda}_2 \left(\frac{M_{c_2}}{M_{c_1}} \right)^4$, $\widetilde{\lambda}_{12} = \lambda_{12}$.

The masses of scalar fields transform as

$$m_1^2 = \tilde{m}_1^2 \left(\frac{\phi_2}{M_{c_2}} \right)^4, \quad (52)$$

$$m_2^2 = \tilde{m}_2^2 \left(\frac{\phi_1}{M_{c_1}} \right)^4. \quad (53)$$

These are the some cases of α 's. One can choice the suitable cases for the relevant physical problems. Not only for stabilizing the Higgs boson, it may useful in representing the dark energy as well.

In the all cases;

1. Even if the action has no scalar field, Ω treats as new scalar fields and transform the gravitational theory to a scalar tensor theory.
2. All kinetic terms in the theory couple to the fields, ϕ_1 and ϕ_2 , via functional couplings $K_1(\phi_1, \phi_2)$, $K_2(\phi_1, \phi_2)$, and $K_{12}(\phi_1, \phi_2)$.
3. In the case 4, the scalar field masses, and vacuum energy become the function of scalar fields, ϕ_1, ϕ_2 , eqs. (52, 53).

5 Conclusion

Conformal transformation transforms ghostly scalar field to the real scalar field in the single scalar field theories. This transformation also helps to reduces the quadratic quantum correction to the scalar field mass, and gives an important way to make Higgs boson, founded in LHC experiments, as a stable particle.

In the multi-scalar field section, the situations contain many specific cases. These situations are directly related to the conformal functional coefficients K_1, K_2 , and K_{12} . Under the suitable choice of these coefficients one may get the desired results. These coefficients can be organized to modeling the inflation, dark energy *etc* as well. The coefficients can be set to the negative values in same cases; for example case 1, and case 2. This is directly related to the characteristic of the real scalar fields. On the other hand, the positive value of the coefficients is related to the geometrical character of the ghostly scalar fields.

For all the values of α 's which we considered; the vacuum energy, and the masses of scalar fields are transform very differently. In the last case, the masses of scalar fields and vacuum energy are related to the scalar fields, ϕ_1 and ϕ_2 .

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